

# True Altitude calculations

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12 February 2013, Varna, Bulgaria

## ABSTRACT

Precise altitude information is necessary for 3D scoring in paragliding competitions. GPS altitude data doesn't respond fast enough to vertical movements and pressure altitude data is distorted by the atmosphere conditions.

It's possible to make analysis of multiple pressure and altitude pairs to calculate how the atmospheric parameters change in time and space. Knowing basis atmosphere parameters – calculating the actual altitude is trivial.

## INTRODUCTION

The recent developments in paragliding competition scoring require precise altitude to be known. Currently all scoring of a paragliding competition is based only on the horizontal position in the flight record. A rare exception is the situation where the task is stopped and pilots are rewarded with few meters ahead for each meter altitude they had at the stop time.

To have fair scoring – precise altitude is necessary. The GPS receivers give very precise altitude, but in static situation. In dynamic situation (when the receiver moves in 3D space) GPS receivers have good horizontal precision but not very good vertical precision. Another downside is the fact that GPS altitude lags behind the actual movement of the receiver – this lag is minimized in the quality receivers but some receivers may lag up to 35 seconds! This can be both advantageous/disadvantageous and therefore is not acceptable for scoring! Barometers give good timing response of the vertical movement (almost no lag) but the reading is very much dependent of current atmospheric conditions. The conditions change in time and in space.

In this study I will investigate the feasibility of a method for combining pressure and GPS data to calculate the true altitude.

## THE PROBLEM

A study [3] shows that GPS receivers can reach vertical accuracy less than 10 m. This accuracy is completely acceptable for paragliding competition scoring (horizontal accuracy is same order) but the GPS receiver needs time to calculate exact altitude. Correlation analysis of different tracklogs shows that different GPS receivers give delays from 2.5 to 35 seconds.

On the figure 1 are shown correlation coefficients between baro-altitude and GPS-altitude when they are sifted against

each-other by step of 1 second. The maximum of the correlation coefficient shows the shift that matches with the delay of the second sequence (GPS). Instrument 2 has delay 3 seconds, but Instrument 1 has delay 35 seconds.

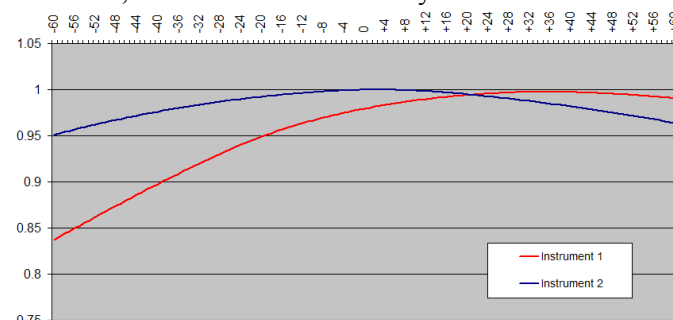


Figure 1

The other problem with GPS altitude is the precision in dynamic environment. Simple experiment with two different receivers side by side in a single flight show absolute errors of a few hundred meters, standard deviation of the difference between the two altitudes more than 40 m and (surprisingly) average difference almost 0. This shows that the GPS receivers are good in measuring vertical distances for long time and by averaging multiple measurements.

Flying instruments have inside GPS module. This module is connected to the internal antenna and through some digital interface connected to the instrument micro-controller/CPU. GPS modules could be divided into 3 types according to altitude information they give:

1. Modules giving the altitude above(below) WGS-84 ellipsoid
2. Modules giving the altitude above(below) WGS-84 geoid
3. Modules giving the altitude above(below) WGS-84 geoid and the geoid height above(below) the ellipsoid

In general the instrument (it's software) has no control over what type of altitude GPS module gives. If the module is one of the first two types, the instrument can store in it's tracklogs only the altitude from the module and (in the best case) report what type this altitude is. If the module is from the third type – the instrument can store in it's tracklogs either geoid or ellipsoid altitude. Again reporting what type of altitude is stored is preferable. According to the IGC standard [4] – the altitude in IGC tracklogs must be altitude above(below) WGS-84 ellipsoid.

Barometric altitude doesn't have the delay problem. Again with a correlation analysis was shown that maximum delay between barometric altitudes of two instruments is 2 seconds. Difference analysis of altitudes from few pairs of tracklogs showed that baro altitudes have very low standard deviation in order of  $10^1$  m and some offset (static error) in order of  $10^2$  m.

Barometric altitude is calculated inside the instrument using ICAO ISA [2] formula:

$$h = \frac{T_0}{L} \left( 1 - \left( \frac{p}{p_0} \right)^{\left( \frac{RL}{gM} \right)} \right) \quad (1)$$

Where:

h – altitude above MSL [m]

T<sub>0</sub> – base temperature at the MLS [°K]

L – temperature laps rate 0.0065 °K/m

p – ambient pressure at altitude h [hPa]

p<sub>0</sub> – base pressure at MSL [hPa]

R – universal gas constant 8.31432 N·m/(mol·K)

g – earth's gravity constant 9.80665 m/s<sup>2</sup>

M – molar mass of the air 0.0289644 kg/mol

The basis of the altitude calculation is the ambient pressure at the aircraft, but it also depends of p<sub>0</sub> and T<sub>0</sub>. On figure 2 are two curves showing how pressure change with the altitude in two different atmospheres. The base parameters p<sub>0</sub> and T<sub>0</sub> are relatively constant during the flight. Paraglider flights (especially in competitions) usually last few hours and are made in zones usually small in global terms (zone max dimension less than 100 km). It could be expected that these two parameters of atmosphere will change slowly during the flight and also they will change gradually with the location of the aircraft in the zone.

Pressure and altitude with different atmosphere parameters

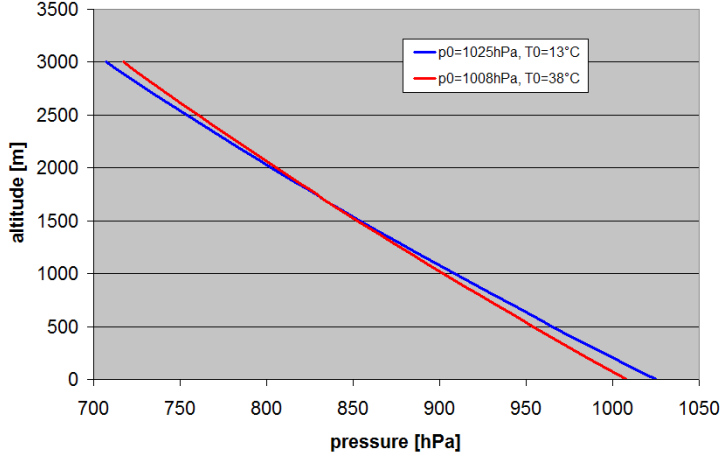


Figure 2

For the purposes of IGC logging altitude is calculated using constant atmosphere parameters (p<sub>0</sub> = 1013.25 hPa, T<sub>0</sub> = 288.15 °K). If I could find how these two parameters of the real atmosphere change in time/space I will be able to calculate precise altitude using the ambient pressure and the p<sub>0</sub>, T<sub>0</sub> pairs at the specific place and time. My expectations are to reach better average and standard deviation of the difference of two calculated “true altitudes” from two different instruments recording a flight side by side.

## METHODS

First I calculate the ambient pressure. From the formula (1) I can evaluate that:

$$p = p_0 \left( 1 - \frac{hL}{T_0} \right)^{\left( \frac{gM}{RL} \right)} \quad (2)$$

To calculate exact pressure is necessary to use the exact constants used in the forward equation (1). This calculation is necessary because in the tracklog there is no pure pressure record. It is also possible different manufacturers to make the forward calculation (1) differently.

Once the ambient pressure is calculated for each tracklog fix – I have multiple pairs of p, h where p is from barometric sensor and h is from GPS receiver. Using two pairs (p, h) I can calculate the atmosphere parameters leading to those two altitude, pressure points.

There is exactly one pressure/altitude line that passes through two points (p<sub>1</sub>, h<sub>1</sub>) and (p<sub>2</sub>, h<sub>2</sub>) show on figure 3. From the system of two equations:

$$\begin{aligned} h_1 &= \frac{T_0}{L} \left( 1 - \left( \frac{p_1}{p_0} \right)^{\left( \frac{RL}{gM} \right)} \right) \\ h_2 &= \frac{T_0}{L} \left( 1 - \left( \frac{p_2}{p_0} \right)^{\left( \frac{RL}{gM} \right)} \right) \end{aligned} \quad (3)$$

I can evaluate the base atmospheric parameters:

$$\begin{aligned} p_0 &= \left( \frac{h_1 p_2 \left( \frac{RL}{gM} \right) - h_2 p_1 \left( \frac{RL}{gM} \right)}{h_1 - h_2} \right)^{\left( \frac{gM}{RL} \right)} \\ T_0 &= \frac{h_1 L}{1 - \left( \frac{p_1}{p_0} \right)^{\left( \frac{RL}{gM} \right)}} = \frac{h_2 L}{1 - \left( \frac{p_2}{p_0} \right)^{\left( \frac{RL}{gM} \right)}} \end{aligned} \quad (4)$$

Graphical representation of two fixes and corresponding p<sub>0</sub>

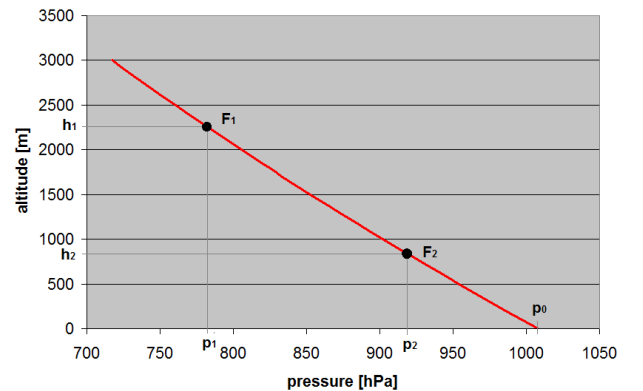


Figure 3

The pairs must have different h. For greater precision good altitude separation is necessary. Error evaluation of the pair vertical separation is shown below for  $p_0$  and  $T_0$ :

$p_0$  errors depending on vertical separation of the points

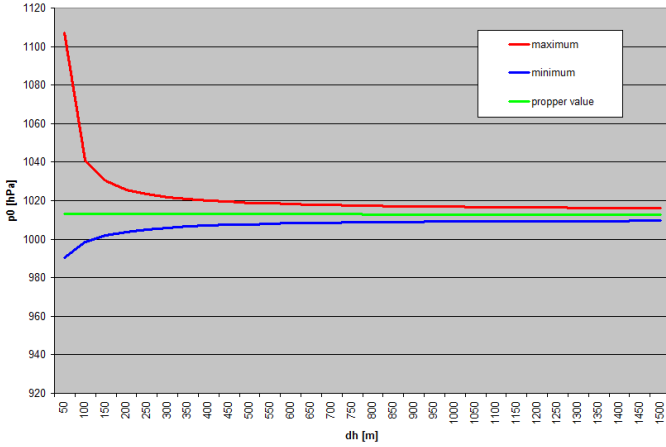


Figure 4

$T_0$  errors dependence of vertical separation of the points

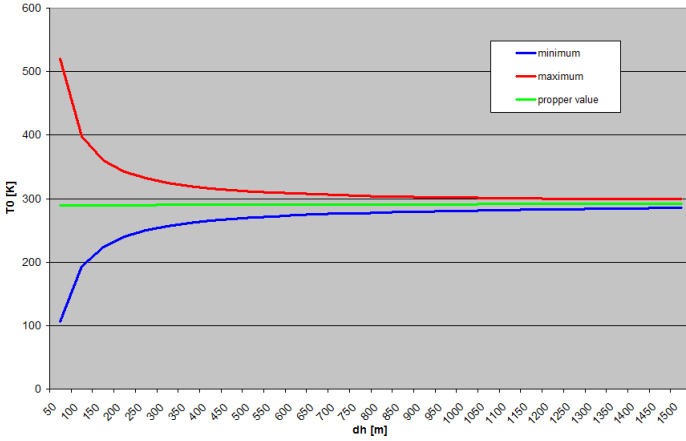


Figure 5

It is clear that points with maximum vertical separation and minimum separation in time and space must be chosen to minimize the errors of the calculated ( $p_0$ ,  $T_0$ ). Having multiple calculated pairs ( $p_0$ ,  $T_0$ ) for multiple points in time/space domain I can find the parameters of a function that will represent the slow changes of the parameters:

$$\begin{aligned} p_0 &= F_p(x, y, t) \\ T_0 &= F_T(x, y, t) \end{aligned} \quad (5)$$

The function parameters I decided to find with linear regression analysis. I didn't choose polynomial regression because the function will have to be extrapolated for the whole time/space domain, but I don't have good pairs around the borders. Example of the regression is shown on figure 6.

For simplification the place is described by single axis. The axis is chosen from the biggest distance between any two points any of the pilots have been. Then each tracklog fix is orthogonally mapped on the chosen axis and the place is represented by single scalar x. Formulas become:

$$\begin{aligned} p_0 &= F_p(x, t) \\ T_0 &= F_T(x, t) \end{aligned} \quad (6)$$

The next step is pre-calculation of the values of the functions for all ( $x$ ,  $t$ ). With the calculated values of  $p_0$  and  $T_0$  precise baro-altitude could be calculated for each fix of each tracklog using equation (1).

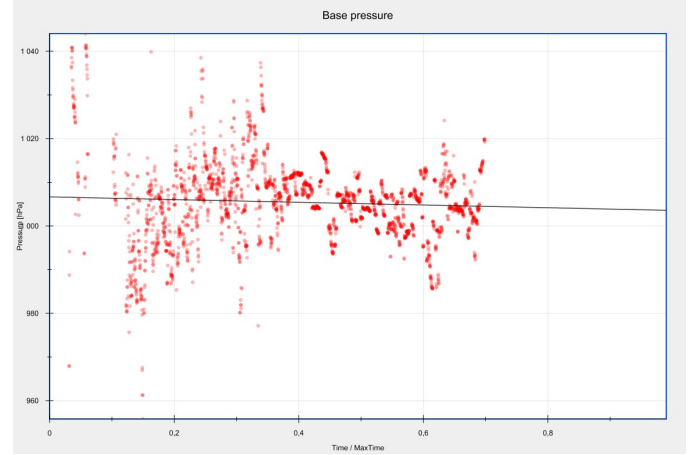


Figure 6

Finally for each tracklog is calculated “pressure offset” value to correct the static errors in the system [pressure sensor → internal pressure reading → ISA pressure altitude calculation → altitude rounding to integer]. The  $O_p$  is calculated as:

$$O_p = \overline{h_{GPS}} - \overline{h_{true}} \quad (7)$$

This value is added to each true altitude fix of the tracklog. As a final result  $Average(h_{GPS} - h_{true}) = 0$

All these calculations I made first in a spreadsheet application. After I reached some results, I made Java program that analyzes all files in given directory and creates another directory with copies of all IGC files found in the first directory but with true altitude instead of baro and GPS altitude. I choose Java because:

- it is easier to understand the code,
- it is platform independent,
- the code could be reused easily,
- there are a lot of libraries for it.

## RESULTS

Results are evaluated from the tracklogs written by the program I developed. In the analyzed tracklog set I put two tracklogs from the same pilot/flight but recorded by two different instruments. For these two instruments I am sure that the actual altitude was the same all the time. The error analysis between the two tracklogs is in the table below.

	<b>Avg</b>	<b>Max</b>	<b>StDev</b>
Baro-altitude	32	36	1.0
GPS-altitude	0	19	2.6
<b>True altitude</b>	<b>0</b>	<b>5</b>	<b>1.0</b>
Eastning	0	15	5.5
Northning	2	15	7.1

Results are in the same order for all competition tasks I was able to analyze. The average difference in the calculated True altitude is 0 as it is for the GPS altitude, but the True altitude has lower maximum error and lower standard deviation. Comparing to position error the results are even better. It was unexpected for me to discover that horizontal accuracy (in which all pilots undoubtedly believe) has errors like showed in the table.

### CONCLUSIONS

Results from the analysis of many competition tasks show that the method described in this document is capable to increase the accuracy of the altitude. The post-processed altitude is with greater accuracy than the primary data and is greater than the horizontal accuracy of the instruments. I can conclude that 3D scoring in paragliding competitions is possible and pilots can trust it. The necessary conditions are all pilots to use IGC flight recorders and the True Altitude calculation method for post-processing to be used.

### FURTHER WORK

In near future I plan to make following improvements of the Java program and other work:

1. To make more tests with different instruments recording the same flight. To evaluate how the algorithm works in various competition environments.
2. To make a test with geodesy-grade precision instrument and compare the results.
3. To include the relative humidity (RH [%]) in all formulas. RH considered to be constant for the flying space/time domain. Given as a command line option, if omitted - auto-detection from the flying ceiling which will be almost always the cloudbase.
4. To implement regression functions with 3 arguments as described above.
5. To implement LatLon2UTM conversion in a given UTM zone. Auto-detection of the most appropriate zone for all fixes to avoid the problems when the flying domain lays in 2 or 3 or 4 UTM zones.
6. To implement UTC offset for all time calculations to avoid the potential problem if the competition is made in Japan or in west USA and UTC date change during the flight is possible.
7. To implement interface to Geographiclib library for retrieving the geoid heights.

8. To implement detection of the instruments known to give altitudes above the geoid.

### RECOMMENDATIONS TO CIVL

1. Make the True Altitude calculations part of FS scoring software
2. Initiate change in the IGC standard of additional field in B records for "Aircraft Pressure" or "Ambient Pressure" (TLC: APR, format: PPPPpp (six digits temperature compensated pressure in hPa)). Stimulate the manufacturers of flying instruments to include this field in the tracklogs their instruments produce.
3. Initiate change in the IGC standard of additional header record for what type of altitude is reported by the GPS module (example: HFALTTYPE: Geoid or Ellipsoid). Stimulate the manufacturers of flying instruments to include this field in the tracklogs their instruments produce.
4. Initiate communication with manufacturers of flying instruments to cover the requirements of the IGC standard in their instruments. Give a date in future (after 1 year for example) as a deadline for covering the standard. After this date only instruments covering the standard should be accepted in Category 1 events. Here "covering the standard" means all of the following:
  - a. To log GPS altitude
  - b. To log barometric altitude
  - c. GPS altitude to be altitude above WGS-84 ellipsoid
  - d. Barometric altitude to be calculated exactly by ICAO ISA formula with the necessary precision and using the exact constants
  - e. To log the barometric altitude without filtering (smoothing) and without any additional processing
  - f. In case GPS module "looses the satellites", barometric logging to continue with timing source - the internal real time clock of the instrument

### REFERENCES

For this document I used information and data from following sources:

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